Write your name here

Surname	Other names

Pearson Edexcel GCE

Centre Number	Candidate Number

Mechanics M5

Advanced/Advanced Subsidiary

Tuesday 21 June 2016 – Morning
Time: 1 hour 30 minutes

Paper Reference
6681/01

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Whenever a numerical value of g is required, take g = 9.8 m s^{-2}, and give your answer to either two significant figures or three significant figures.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

1. [*In this question*, **i** *and* **j** *are perpendicular unit vectors in a horizontal plane.*]

A bead P of mass 0.4 kg is threaded on a smooth straight horizontal wire. The wire lies along the line with vector equation $\mathbf{r} = (\mathbf{i} + 2\mathbf{j}) + \lambda(-2\mathbf{i} + 3\mathbf{j})$. The bead is initially at rest at the point A with position vector $(-\mathbf{i} + 5\mathbf{j})$ m. A constant horizontal force $(0.5\mathbf{i} + \mathbf{j})$ N acts on P and moves it along the wire to the point B. At B the speed of P is 5 m s^{-1}.

Find the position vector of B.

(Total 7 marks)

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2. A particle P is moving in a plane. At time t seconds the position vector of P is $\mathbf{r}$ metres and the velocity of P is $\mathbf{v}$ m s^{-1}. When $t = \frac{\pi}{2}$, P is instantaneously at rest at the point with position vector $(\mathbf{i} - \mathbf{j})$ m.

Given that $\mathbf{r}$ satisfies the differential equation

$$\frac{\mathrm{d}^2\mathbf{r}}{\mathrm{d}t^2} + 4\mathbf{r} = (3 \sin t)\,\mathbf{i},$$

find $\mathbf{v}$ in terms of t.

(Total 13 marks)

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3. Three forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ act on a rigid body at the points with position vectors $\mathbf{r}_1$, $\mathbf{r}_2$ and $\mathbf{r}_3$ respectively, where

$\mathbf{F}_1 = (2\mathbf{j} - \mathbf{k})$ N $\quad \mathbf{F}_2 = (\mathbf{i} + \mathbf{k})$ N $\quad \mathbf{F}_3 = (\mathbf{i} + \mathbf{j})$ N

$\mathbf{r}_1 = (4\mathbf{j} - \mathbf{k})$ m $\quad \mathbf{r}_2 = (2\mathbf{i} + \mathbf{k})$ m $\quad \mathbf{r}_3 = (3\mathbf{i} + \mathbf{j} + \mathbf{k})$ m

The system of the three forces is equivalent to a single force $\mathbf{R}$ acting through the point with position vector $(\mathbf{i} - \mathbf{j} + \mathbf{k})$ m, together with a couple of moment $\mathbf{G}$.

(*a*) Find $\mathbf{R}$.

(2)

(*b*) Find $\mathbf{G}$.

(9)

(Total 11 marks)

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4. Find, using integration, the moment of inertia of a uniform cylindrical shell of radius r, height h and mass M, about a diameter of one end.

(Total 10 marks)

5.

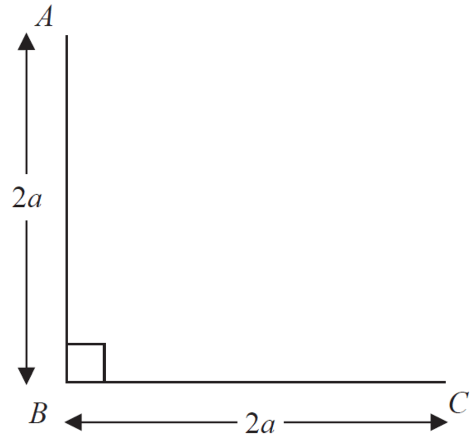

Figure 1

A uniform piece of wire ABC, of mass $2m$ and length $4a$, is bent into two straight equal portions, AB and BC, which are at right angles to each other, as shown in Figure 1. The wire rotates freely in a vertical plane about a fixed smooth horizontal axis L which passes through A and is perpendicular to the plane of the wire.

(a) Show that the moment of inertia of the wire about L is $\dfrac{20ma^2}{3}$.

(3)

(b) By writing down an equation of rotational motion for the wire as it rotates about L, find the period of small oscillations of the wire about its position of stable equilibrium.

(8)

(Total 11 marks)

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6. A firework rocket, excluding its fuel, has mass m_0 kg. The rocket moves vertically upwards by ejecting burnt fuel vertically downwards with constant speed u m s^{-1}, $u > 24.5$, relative to the rocket. The rocket starts from rest on the ground at time $t = 0$. At time t seconds, $t \leq 2$, the speed of the rocket is v m s^{-1} and the mass of the rocket including its fuel is $m_0(5 - 2t)$ kg. It is assumed that air resistance is negligible and the acceleration due to gravity is constant.

(*a*) Show that, for $t \leq 2$,

$$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{2u}{5-2t} - 9.8 .$$

(6)

(*b*) Find the speed of the rocket at the instant when all of its fuel has been burnt.

(6)

(Total 12 marks)

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7. A uniform square lamina $PQRS$, of mass m and side $2a$, is free to rotate about a fixed smooth horizontal axis which passes through P and Q. The lamina hangs at rest in a vertical plane with SR below PQ and is given a horizontal impulse of magnitude J at the midpoint of SR. The impulse is perpendicular to SR.

(*a*) Find the initial angular speed of the lamina.

(3)

(*b*) Find the magnitude of the angular deceleration of the lamina at the instant when the lamina has turned through $\frac{\pi}{6}$ radians.

(3)

(*c*) Find the magnitude of the component of the force exerted on the lamina by the axis, in a direction perpendicular to the lamina, at the instant when the lamina has turned through $\frac{\pi}{6}$ radians.

(5)

(Total 11 marks)

TOTAL FOR PAPER: 75 MARKS

Question Number	Scheme	Marks
1.	Let pv of $B = a\mathbf{i} + b\mathbf{j}$	
	$(0.5\mathbf{i}+\mathbf{j}).((a+1)\mathbf{i}+(b-5)\mathbf{j}) = \frac{1}{2}\text{x}0.4\text{x}5^2$	M1 A1
	$a + 2b = 19$	M1 A1
	$a = 1 - 2\lambda;\ b = 2 + 3\lambda$	B1
	solving for a and b	M1
	pv of $B = (-6\mathbf{i} + 12.5\mathbf{j})$ m	A1 **7**
	OR	
	$(0.5\mathbf{i}+\mathbf{j}).(-2\lambda\mathbf{i}+3\lambda\mathbf{j}) = \frac{1}{2}\text{x}0.4\text{x}5^2$	M1 A1
	$-\lambda + 3\lambda = 5$	M1 A1
	$\lambda = \frac{5}{2}$	B1
	pv of $B = (-\mathbf{i}+5\mathbf{j}) + \frac{5}{2}(-2\mathbf{i}+3\mathbf{j})$	M1
	$= (-6\mathbf{i} + 12.5\mathbf{j})$ m	A1 **7**
	<u>Alternative using forces and acceleration</u>	
	Resolving along the wire: $(0.5\mathbf{i}+\mathbf{j}).\frac{1}{\sqrt{13}}(-2\mathbf{i}+3\mathbf{j}) = 0.4a$	M1
	$\frac{5}{\sqrt{13}} = a$	A1
	$v^2 = u^2 + 2as:\quad 5^2 = 2\times\frac{5}{\sqrt{13}}s$	M1
	$\frac{5\sqrt{13}}{2} = s$	A1
	$\mathbf{AB} = \frac{5}{2}(-2\mathbf{i}+3\mathbf{j}) = (-5\mathbf{i}+7.5\mathbf{j})$	B1
	pv of $B = (-\mathbf{i}+5\mathbf{j}) + (-5\mathbf{i}+7.5\mathbf{j})$	M1
	$= (-6\mathbf{i}+12.5\mathbf{j})$	A1 **7**
	Notes	
	First M1 for attempt at using work-energy principle, with usual rules First A1 for a correct unprocessed equation Second M1 for producing an equation in a and b (seen or implied) Second A1 for a correct equation B1 for $a = 1 - 2\lambda$ and $b = 2 + 3\lambda$ seen or implied Third M1 for solving for a and b Third A1 for correct answer (must be a <u>vector</u>)	

Alt	First M1 for resolving along the wire, with usual rules First A1 for a correct acceleration seen or implied Second M1 for a complete method, using *suvat* or calculus, to find the distance along the wire Second A1 for a correct distance B1 for **AB** = (-5**i** + 7.5**j**) seen or implied Third M1 for finding the answer Third A1 for correct answer (must be a <u>vector</u>)	

Question Number	Scheme	Marks
2		
	$\lambda^2 + 4 = 0$	M1
	$\mathbf{r} = \mathbf{A}\cos 2t + \mathbf{B}\sin 2t$	A1
	PI: $\mathbf{r} = \mathbf{c}\sin t + \mathbf{d}\cos t$	B1
	$\mathbf{r}' = \mathbf{c}\cos t - \mathbf{d}\sin t$	M1
	$\mathbf{r}'' = -\mathbf{c}\sin t - \mathbf{d}\cos t$	A1
	$-\mathbf{c}\sin t - \mathbf{d}\cos t + 4(\mathbf{c}\sin t + \mathbf{d}\cos t) \equiv 3\sin t\ \mathbf{i}$	M1
	$\mathbf{c} = \mathbf{i};\ \mathbf{d} = \mathbf{0}$	M1
	GS is $\mathbf{r} = \mathbf{A}\cos 2t + \mathbf{B}\sin 2t + \mathbf{i}\sin t$	A1
	When $t = \frac{\pi}{2}, \mathbf{r} = \mathbf{i} - \mathbf{j} \Rightarrow \mathbf{A} = \mathbf{j}$	M1 A1
	$\mathbf{v} = -2\mathbf{A}\sin 2t + 2\mathbf{B}\cos 2t + \mathbf{i}\cos t$	M1
	When $t = \frac{\pi}{2}, \mathbf{v} = \mathbf{0} \Rightarrow \mathbf{B} = \mathbf{0}$	A1
	$\mathbf{r} = \mathbf{i}\sin t + \mathbf{j}\cos 2t$	
	$\mathbf{v} = \mathbf{i}\cos t - 2\mathbf{j}\sin 2t$	A1
		13
	Notes	
	First M1 for auxiliary equation First A1 for correct CF (condone omission of **r** =) B1 for correct PI (they may realise **d** = **0** which is fine) (condone omission of **r** =) Second M1 for differentiating their PI Second A1 for correct 2nd derivative Third M1 for substituting into the DE Fourth M1 for equating coeffs of sin*t* and cos *t* and finding **c** and **d** Third A1 for correct GS with **r** = Fifth M1 for using conditions to find **A** Fourth A1 for **A**= **j** Sixth M1 for differentiating **r** to give **v** Fifth A1 for **B** = **0** Sixth A1 for **v = i**cos*t* – 2**j**sin2*t*	

Question Number	Scheme	Marks
3(a)	$\mathbf{R} = (2\mathbf{j} - \mathbf{k}) + (\mathbf{i} + \mathbf{k}) + (\mathbf{i} + \mathbf{j}) = (2\mathbf{i} + 3\mathbf{j})\text{N}$	M1 A1 (2)
3(b)	Moments about O: $(4\mathbf{j} - \mathbf{k})\times(2\mathbf{j} - \mathbf{k}) + (2\mathbf{i} + \mathbf{k})\times(\mathbf{i} + \mathbf{k}) + (3\mathbf{i} + \mathbf{j} + \mathbf{k})\times(\mathbf{i} + \mathbf{j})$ $= \quad -2\mathbf{i} \quad + \quad -\mathbf{j} \quad + \quad -\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ $= \quad (-3\mathbf{i} + 2\mathbf{k})$ $(\mathbf{i} - \mathbf{j} + \mathbf{k}) \times (2\mathbf{i} + 3\mathbf{j}) + \mathbf{G} = (-3\mathbf{i} + 2\mathbf{k})$ $(-3\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}) + \mathbf{G} = (-3\mathbf{i} + 2\mathbf{k})$ $\mathbf{G} = (-2\mathbf{j} - 3\mathbf{k})$ Nm	M1 A3 M1 **A2 ft** A1 A1 (9) **11**
3(b) Alt	Moments about (1, -1, 1): $(-\mathbf{i} + 5\mathbf{j} - 2\mathbf{k})\times(2\mathbf{j} - \mathbf{k}) + (\mathbf{i} + \mathbf{j})\times(\mathbf{i} + \mathbf{k}) + (2\mathbf{i} + 2\mathbf{j})\times(\mathbf{i} + \mathbf{j})$ $= \quad -\mathbf{i} - \mathbf{j} - 2\mathbf{k} \quad + \quad \mathbf{i} - \mathbf{j} - \mathbf{k} \quad + \quad \mathbf{0}$ $= \quad -2\mathbf{j} - 3\mathbf{k}$ Comparing the 2 systems: $\mathbf{0} + \mathbf{G} = (-2\mathbf{j} - 3\mathbf{k})$ Nm $\mathbf{G} = (-2\mathbf{j} - 3\mathbf{k})$Nm	M1A2 A3 M1 **A1 ft** A1 (9)
	Notes	
3(a)	M1 for adding the 3 forces together A1 for (2**i** + 3**j**)	
3(b)	First M1 consistent use of **r** x **F**, with correct no. of terms First A3 for 3 correct vector products, -1 for each incorrect <u>product</u> (A1A1A0) Second M1 for comparing the rotational effect of the 2 systems *about O*, to give an equation: Their $\Sigma\, \mathbf{r} \times \mathbf{F} = \mathbf{G} + (\mathbf{i} - \mathbf{j} + \mathbf{k}) \times$ their **R** with correct terms (M0 if term missing) Second **A2 ft,** for the equation ft on their **R** and their $\Sigma\, \mathbf{r} \times \mathbf{F}$, but no products need to be evaluated. Sixth A1 for a correct equation with all products evaluated Seventh A1 for the answer. Units not needed.	
3(b) Alt	First M1 consistent use of $\Sigma\, \mathbf{r} \times \mathbf{F}$, with correct no. of terms, with **r** relative to (1, -1 , 1) First A2 for correct terms, -1 each incorrect vector First A3 for 3 correct vector products, -1 for each incorrect <u>product</u> Second M1 for comparing the rotational effect of the 2 systems *about* (1, -1 , 1),to give an equation: $\mathbf{0} + \mathbf{G} =$ their $\Sigma\, \mathbf{r} \times \mathbf{F}$ Sixth **A1, ft** on their $\Sigma\, \mathbf{r} \times \mathbf{F}$, for a correct equation Seventh A1 for the answer. Units not needed.	

Question Number	Scheme	Marks
4.	$dA = 2prdx$	M1
	$dm = 2prdx.\dfrac{M}{2prh}$	M1 A1 (r)
	$(=\dfrac{Mdx}{h})$	
	$\dfrac{1}{2}dmr^2$	B1
	$dI = \dfrac{1}{2}dmr^2 + dmx^2$	M1 A1
	$= \dfrac{Mdx}{2h}(r^2 + 2x^2)$	A1
	$I = \int_0^h \dfrac{M}{2h}(r^2 + 2x^2)\,\mathrm{d}x$	**DM1**
	$= \dfrac{M}{2h}\left[(r^2 x + \tfrac{2}{3}x^3\right]_0^h$	A1
	$= \dfrac{M}{6}(3r^2 + 2h^2)$	A1 **10**
	Notes	
	First M1 for area of hoop (element) Second M1 for finding the mass of their element by multiplying by the mass per unit area (or by a calculated r) First A1 for a correct mass per unit area (appropriate r) First B1 for correct MI about diameter of hoop Third M1 for use of parallel axes Second A1 for correct in terms of dm Third A1 for correct MI in terms of x Fourth **DM1** dependent on third M1 for integrating Fourth A1 for a correct expression with correct limits Fifth A1 for a correct answer in any form	
	N.B. The first 8 marks are available for misreads of solid cylinder or cylindrical shell with end(s).	

Question Number	Scheme	Marks
5(a)	$I_L = \frac{4}{3}ma^2 + \frac{1}{3}ma^2 + m(a^2 + (2a)^2)$ $= \frac{20}{3}ma^2$ PRINTED ANSWER	M1 A1 A1 (3)
(b)	$m\begin{pmatrix}0\\a\end{pmatrix} + m\begin{pmatrix}a\\0\end{pmatrix} = 2m\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix} \Rightarrow \bar{x} = \bar{y} = \frac{a}{2}$ $AG = \frac{a}{2}\sqrt{1^2+3^2} = \frac{a}{2}\sqrt{10}$ $M(A),\quad 2mg\frac{a}{2}\sqrt{10}\sin q = -\frac{20}{3}ma^2\ \ddot{q}$ $-\frac{3g\sqrt{10}}{20a}q = \ddot{q}$, for small q $T = 2p\sqrt{\frac{2a\sqrt{10}}{3g}}$	M1 A1 M1 A1 A1 M1 **DM**1 A1 (8) **11**
	Notes	
5(a)	First M1 for a complete method to find MI with correct no. of terms First A1 for a correct expression Second A1 for a correct PRINTED ANSWER. M0 for $4/3ma^2 + 4/3ma^2 + m(2a)^2$	
5(b)	First M1 for attempt to find *both coordinates* of the CM First A1 for correct position (They could find this by inspection) Second M1 for moments about *A*, with correct no. of terms, and usual rules, in particular the RHS must be dimensionally correct. Second and third A marks for a correct general equation with θ the angle between *AG* and the vertical, A1 for each side. Third M1 for use of small angle approximation and putting into SHM form. **N.B.** Not available if θ is not the angle between *AG* and the vertical Fourth **DM**1, dependent on third M1, for $\frac{2p}{w}$ Fourth A1 for a correct answer in any form	

Question Number	Scheme	Marks
6(a)	$-mg\delta t = (m+\delta m)(v+\delta v)+(-\delta m)(v-u)-mv$ $-mg\delta t = m\delta v + u\delta m$ $-g = \frac{\mathrm{d}v}{\mathrm{d}t}+\frac{u}{m}\frac{\mathrm{d}m}{\mathrm{d}t}$ $m = m_0(5-2t) \Rightarrow \frac{\mathrm{d}m}{\mathrm{d}t} = -2m_0$ $-9.8-\frac{u}{m_0(5-2t)}.(-2m_0)=\frac{\mathrm{d}v}{\mathrm{d}t}$ $\frac{2u}{(5-2t)}-9.8=\frac{\mathrm{d}v}{\mathrm{d}t}$ PRINTED	M1 A2 B1 M1 A1 (6)
(b)	$v=-u\ln(5-2t)-9.8t(+C)$ $t=0, v=0 \Rightarrow C = u\ln 5$ $v=u\ln\frac{5}{(5-2t)}-9.8t$ $m_0 = m_0(5-2t) \Rightarrow t=2$ $v = u\ln 5 - 19.6$ (m s^{-1})	M1 A1 M1 A1 M1 A1 (6) **12**
	Notes	
6(a)	First M1 for a general impulse-momentum equation (correct number of terms, excluding any $\delta m\delta v$ terms) First A2 for a correct equation B1 for $m = m_0(5-2t) \Rightarrow \frac{\mathrm{d}m}{\mathrm{d}t} = -2m_0$ Third M1 for substituting for dm/dt to produce an equation in v and t only. Third A1 for PRINTED ANSWER (need 9.8 not g)	
6(b)	First M1 for separating the variables and integrating First A1 for a correct solution (without C) Second M1 for using conditions (or lower limits) Second A1 for a correct particular solution in t Third M1 for putting $m = m_0$ and solving for t. Use of $t = 2$ can imply this M mark Third A1 for correct answer (allow $2g$ instead of 19.6)	

Question Number	Scheme	Marks
7(a)	$J.2a = \frac{4}{3}ma^2 w$ $w = \frac{3J}{2ma}$	M1 A1 A1 (3)
(b)	$mga\sin 30^\circ = -\frac{4}{3}ma^2\ddot{q}$ $\lvert\ddot{q}\rvert = \frac{3g}{8a}$	M1 A1 A1 (3)
(c)	$X - mg\cos 60^\circ = ma\ddot{q}$ $X - \frac{1}{2}mg = m(-\frac{3g}{8a})a$ $X = \frac{mg}{8}$	M1 A1 A1 M1 A1 (5) **11**
	Notes	
7(a)	First M1 for moment of impulse = Gain in Angular Momentum with all terms dimensionally correct First A1 for a correct equation Second A1 for correct answer	
7(b)	First M1 for taking moments about the axis with all terms dimensionally correct First A1 for a correct equation (q need not be substituted) Second A1 for a correct positive answer **Alternative:** First M1 for differentiating an energy equation with all terms dimensionally correct First A1 for a correct equation Second A1 for a correct positive answer	
7(c)	First M1 for resolving perpendicular to the lamina, with correct no. of terms, with all terms dimensionally correct First and Second A1's for a correct equation (q need not be substituted) Second M1 for substituting for (must be dimensionally correct) $\ddot{q}$ Third A1 for answer	